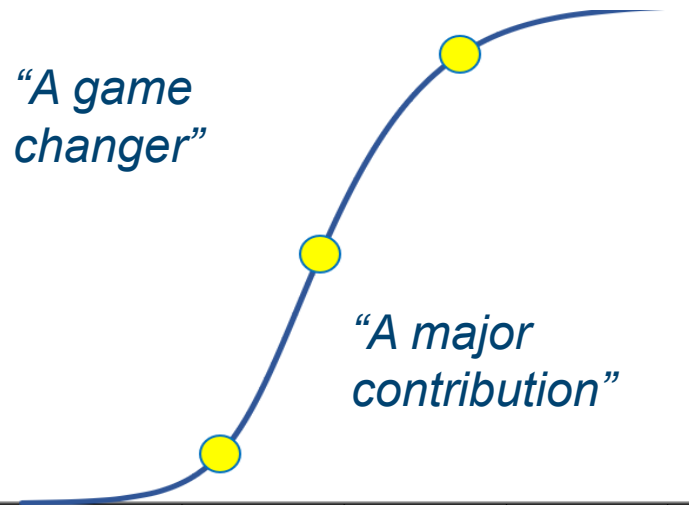


# The Metalog Distributions: Future of Risk Management Quantification

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tomk@keelinreeds.com  
[www.metalogs.org](http://www.metalogs.org)

*Prepared for Risk Awareness Week 2022*



*“A game changer”*

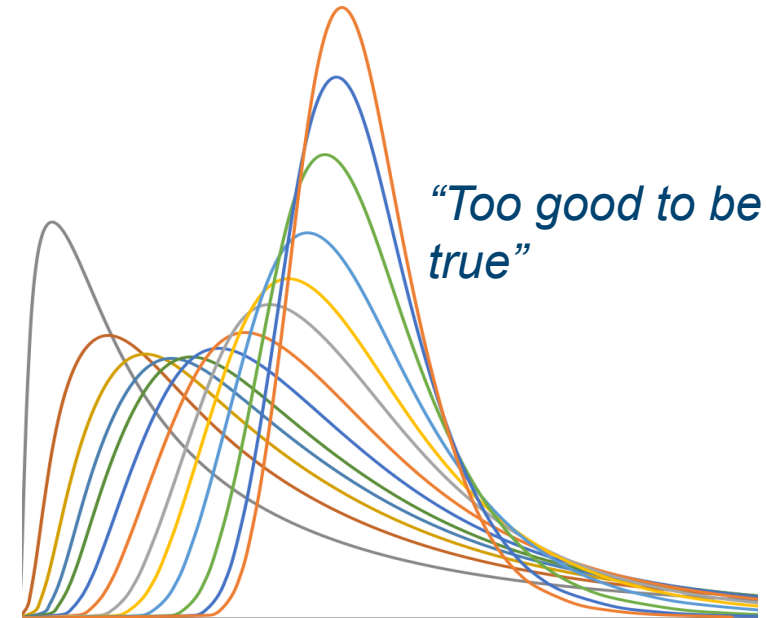
*“A major contribution”*

*“A Tour De Force” -- Ron Howard  
(Stanford Professor, Co-inventor of Decision Analysis)*

*“Metalogs are like a miracle”*

*“So clean and simple”*

*“Can’t believe no one thought of this before”*



*“Too good to be true”*

*“I believe!” -- Harry Markowitz  
(Nobel Laureate)*

*“A scientific breakthrough is like a ladder: suddenly there’s more low-hanging fruit” -- Sam Savage*

*(Adjunct Stanford Professor)*

# The Metalog Distributions: Future of Risk Management Quantification

## *RAW 2022 Agenda*

~ minutes

- |  |    |
|--|----|
| 1. Introduction: Why metalog distributions for risk management | 10 |
| 2. Practical Uses  | 35 |
| • Representing empirical data                                  |    |
| • Representing simulated data                                  |    |
| • Representing expert-elicited data                            |    |
| • Combining the opinion of multiple experts                    |    |
| • Bayesian updating with new data                              |    |
| 3. Using metalog distributions in your models                  | 15 |
| • Bringing metalogs into your modelling environment            |    |
| • Simulating with metalogs                                     |    |
| • Retrieving the metalog equations                             |    |

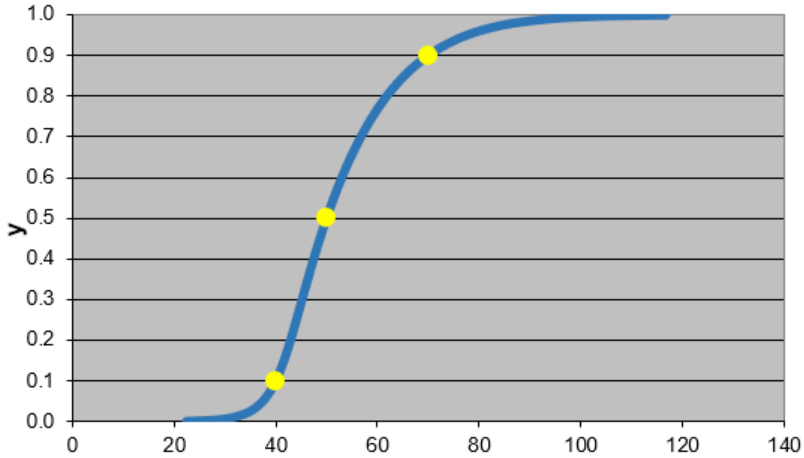
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60

# Metalog distributions are likely your best starting point for fitting continuous distributions to data.

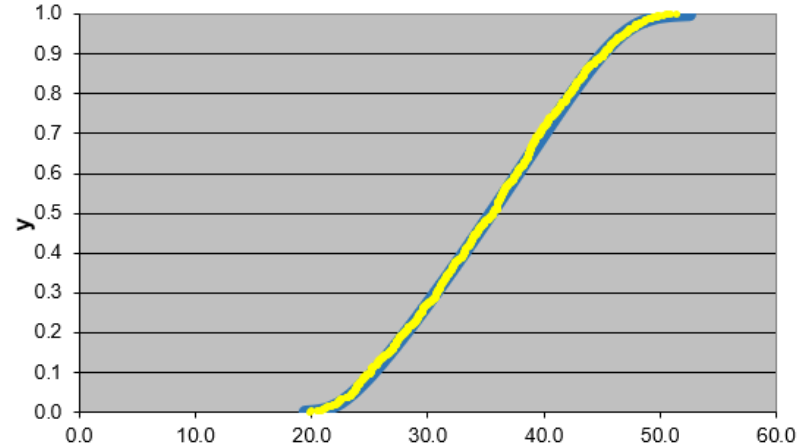
## Expert Elicitation Data

### market size CDF



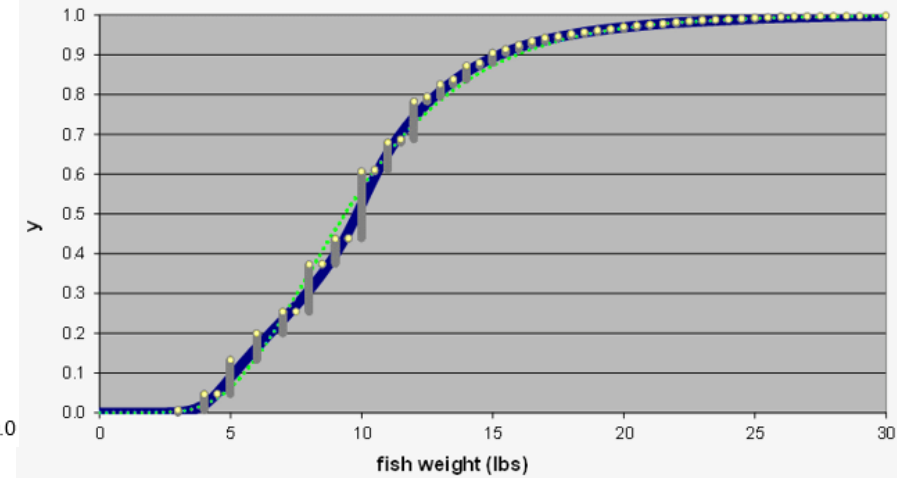
## Simulation Data

### portfolio simulation CDF

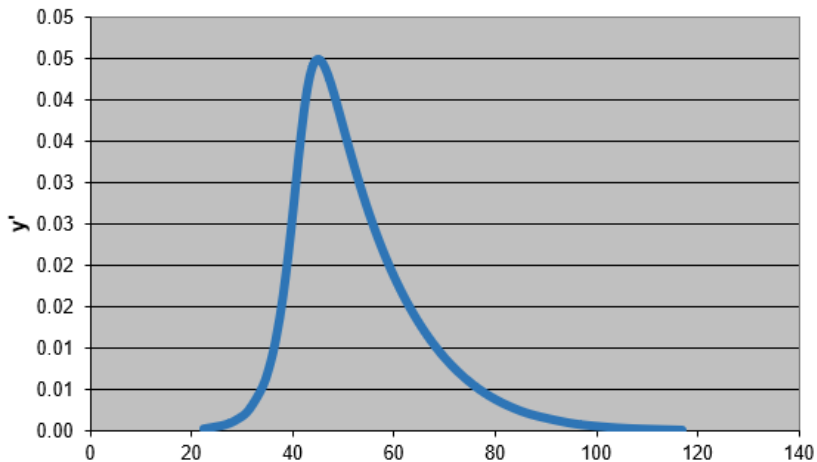


## Empirical Data

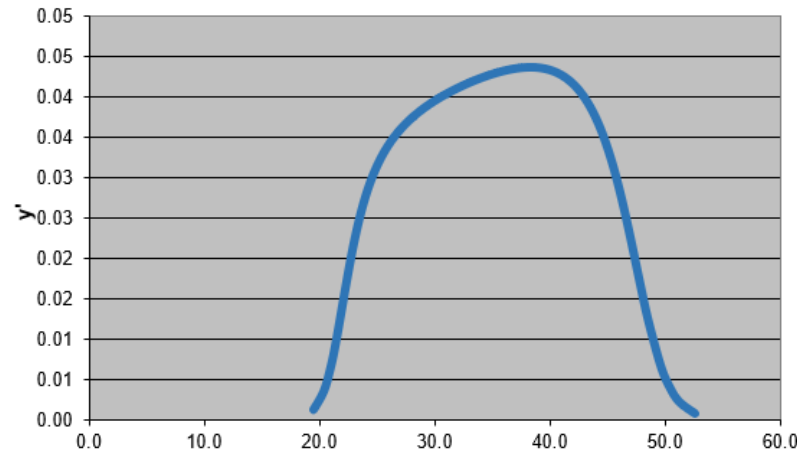
### Cumulative Probability



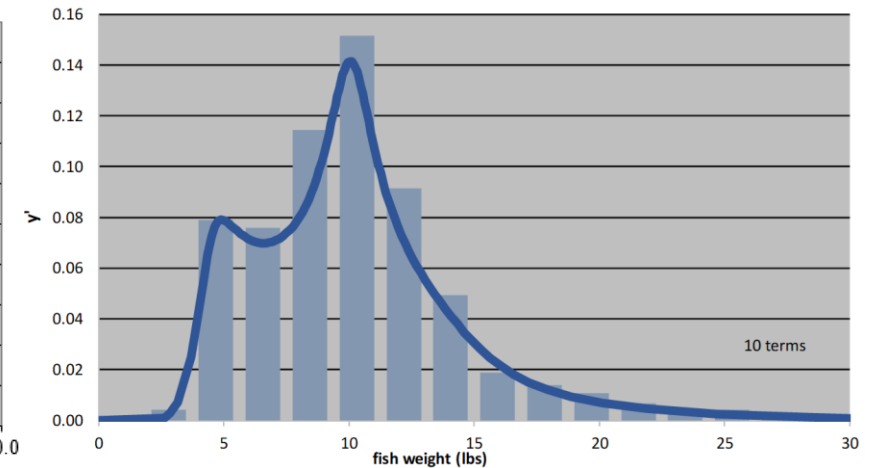
### market size PDF



### portfolio simulation PDF



### Probability Density



# Why opt for metalog distributions instead of classical distributions?

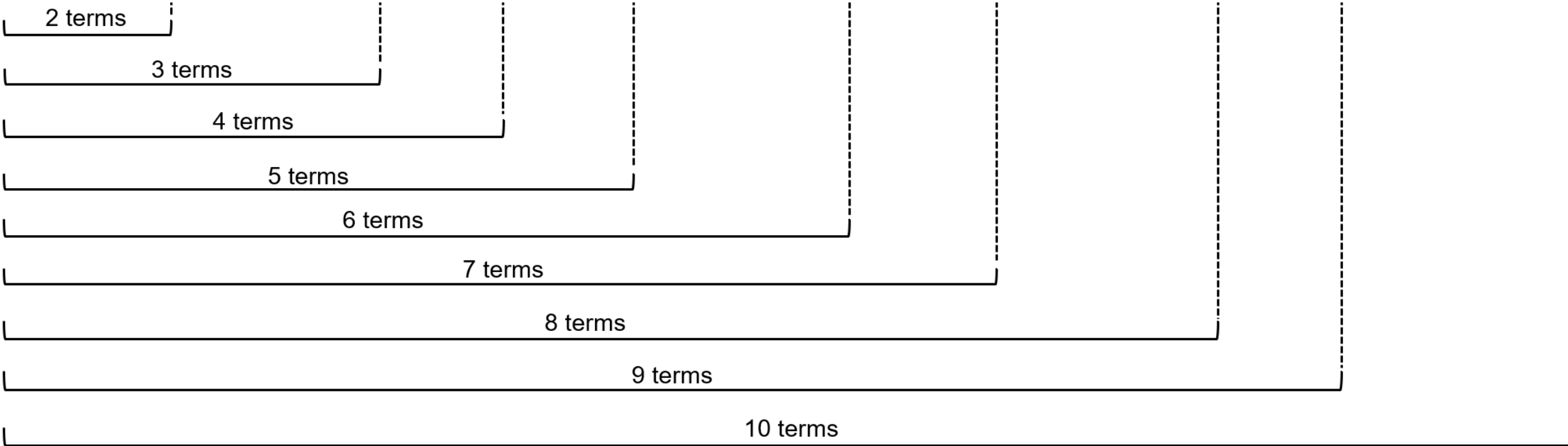
1. Virtually unlimited shape flexibility
  - Like a Taylor series, metalogs can have any number of terms
2. Parameterized by data (least squares fit)
3. Choice of boundedness (unbounded, semi-bounded, or bounded)
4. Simple closed-form equations
5. Easy to simulate -- closed-form quantile function ( $F^{-1}$ )

***Virtually unlimited applicability in any field of human endeavor that uses data.***

**Like Taylor series, metalog distributions may have any number of terms.**

$x = M(y) =$

$$a_1 + a_2 \ln\left(\frac{y}{1-y}\right) + a_3(y - 0.5) \ln\left(\frac{y}{1-y}\right) + a_4(y - 0.5) + a_5(y - 0.5)^2 + a_6(y - 0.5)^2 \ln\left(\frac{y}{1-y}\right) + a_7(y - 0.5)^3 + a_8(y - 0.5)^3 \ln\left(\frac{y}{1-y}\right) + a_9(y - 0.5)^4 + a_{10}(y - 0.5)^4 \ln\left(\frac{y}{1-y}\right)$$



***Each additional term adds shape flexibility for a tighter fit to your data.***

# Metalog distributions are an alternative to classical distributions.

## classical distributions

- Normal
- Exponential
- Gamma
- Uniform
- Student t
- F
- Beta
- Beta Prime
- Chi Squared
- Inverse Chi Squared
- Inverse Gamma
- Pearson IV

... Wikipedia lists ~115 named continuous classical distributions

## metalog distributions

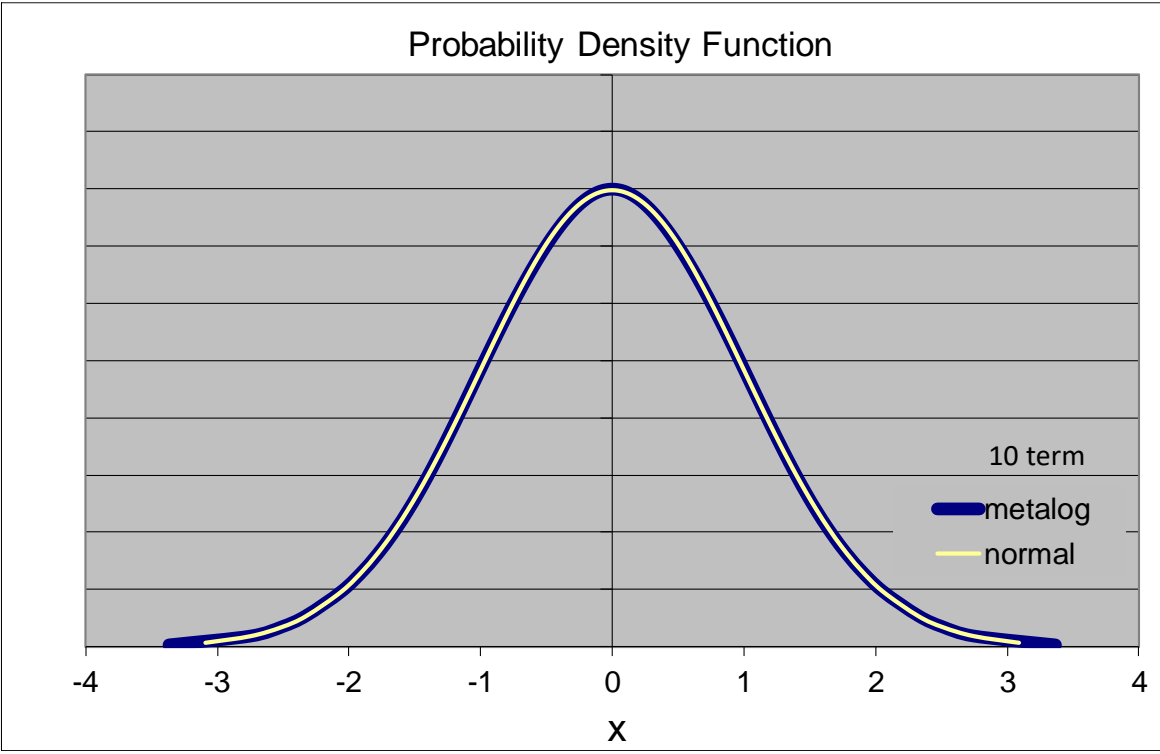
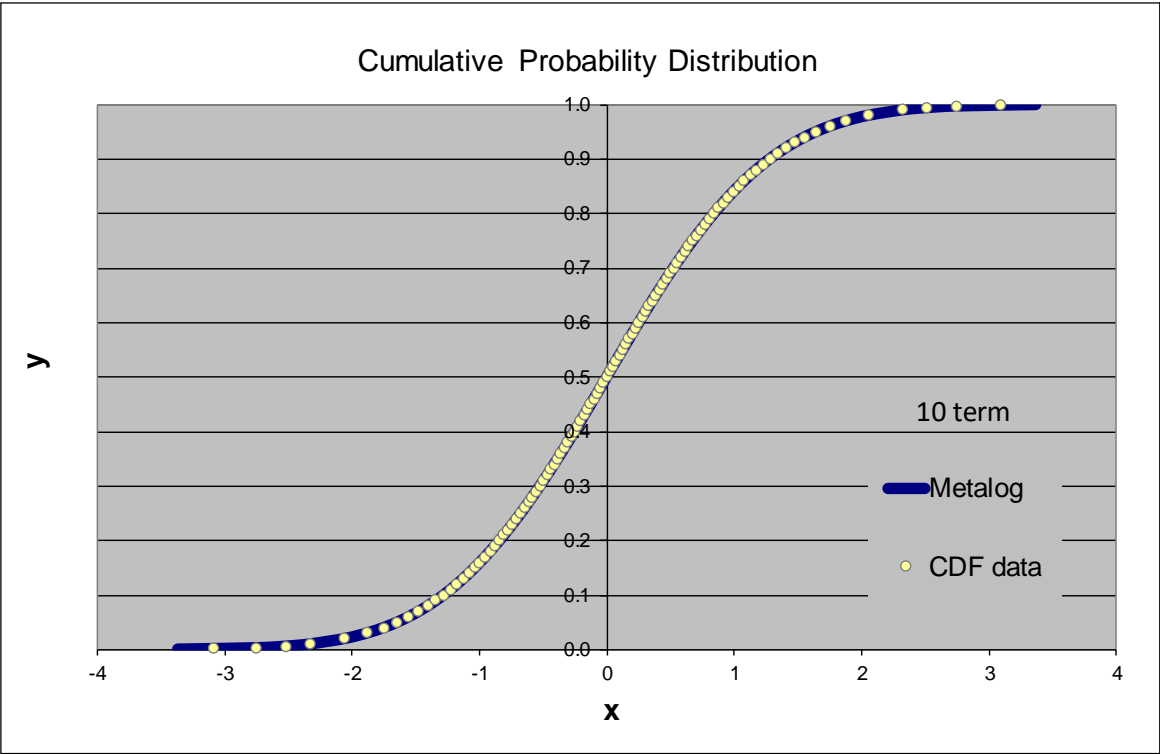
- 15 unbounded distributions
- 15 semi-bounded-low distributions
- 15 semi-bounded-high distributions
- 15 bounded distributions

60 practical metalog distributions

***Moreover, the single metalog equation can represent virtually any distribution.***

# Regardless of shape of your data, there's a metalog distribution to match it.

Metalog approximations of a Standard Normal Distribution ( $\mu = 0, \sigma = 1$ )



# Metalogs are infinitely flexible and can take on any shape.

Theorem: Any probability distribution with a continuous quantile function can be approximated arbitrarily closely by a metalog.

Proof: Any continuous function can be approximated arbitrarily closely by a polynomial. The metalog contains a polynomial. (Weierstrass approximation theorem special case.)

Observations:

- A single set of (metalog) equations can represent any distribution.
- Potentially obsolete questions:
  - “What probability distribution is best for this application?”
  - “How do we fit it to our data?”
- Practical limitation: distributions with spikes, discontinuities, and/or extremely fat tails

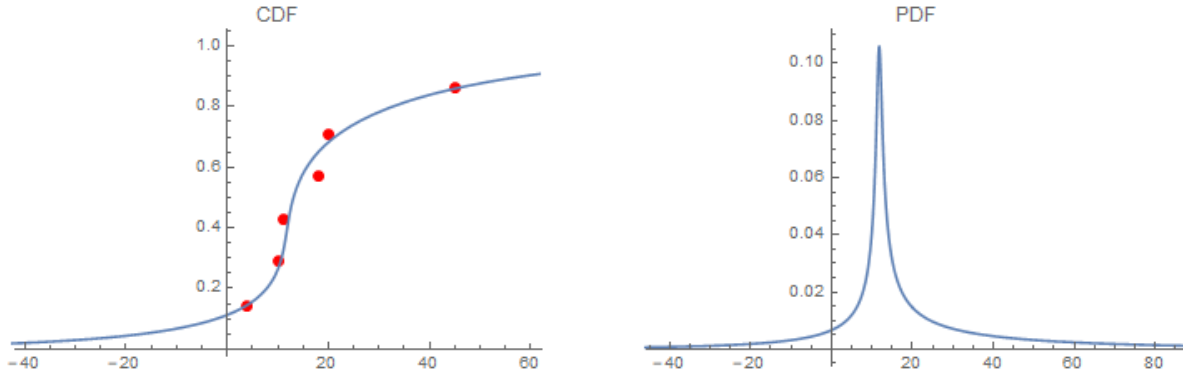


# Limitations and Considerations

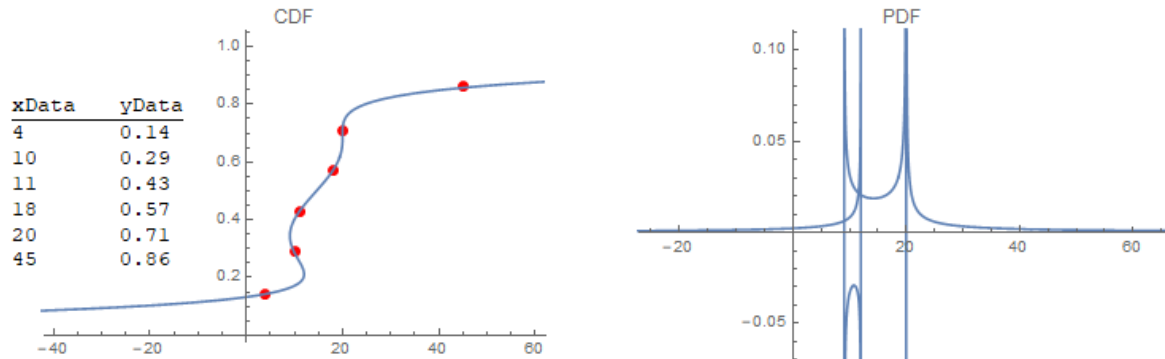
## Feasibility

Example: expert assessment

4-term metalog

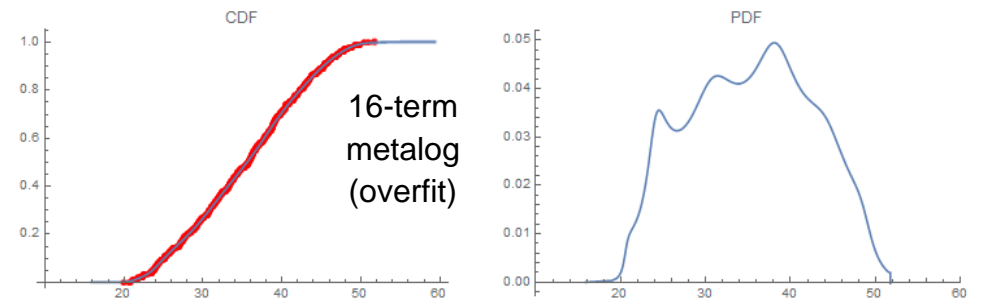
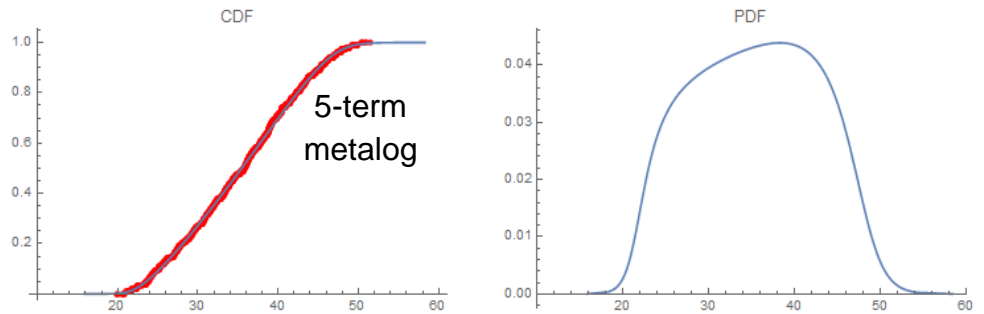
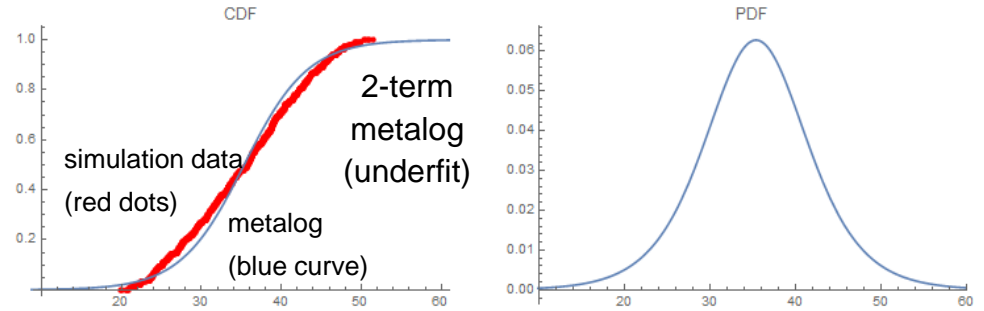


6-terms (infeasible)



## Underfitting and Overfitting

Example: bidding decision simulation, portfolio value of 259 assets



# More information is freely available.

The diagram illustrates the interconnectedness of information sources. It features four main panels:

- YouTube Channel:** "The Metalog Distributions" with 32 subscribers. It features a video titled "Introduction to the Metalog Distributions" and two specific videos: "FLASH Metalogs" (6:57) and "Overview of the Metalogs" (18:01). A URL is provided: <https://www.youtube.com/channel/UCyHZ5neKhV1mSsedzDBoqyA>.
- Wikipedia Article:** "Metalog distribution" from Wikipedia, the free encyclopedia. It includes a link to the full article: [https://en.wikipedia.org/wiki/Metalog\\_distribution](https://en.wikipedia.org/wiki/Metalog_distribution).
- Website:** "The Metalog Distributions" website at [www.metalogs.org](http://www.metalogs.org). The site features a navigation menu with options like Home, Equations, Fit to Data, Shape Flexibility, Moments, Software, Applications, Videos, Publications, and About.
- Journal Article(s):** Published in "DECISION ANALYSIS", Vol. 13, No. 4, December 2016, pp. 243-277. The article is available at <https://pubsonline.informs.org/doi/abs/10.1287/deca.2016.0338>. The journal is published by INFORMS.

Yellow arrows indicate the flow of information: from the YouTube channel to the Wikipedia article, from the website to the journal article, and from the YouTube channel to the journal article.

# Available software resources are updated regularly on the website.



<http://metalogdistributions.com/software.html>

## Software Resources

Software implementations of metalog distributions that are known to us are listed below. There are likely others of which we are unaware.

[Excel Workbooks](#). We offer the complete metalog system (including [metalog\\_panel](#)) within free, convenient Excel workbooks programmed by the author of this website. The worksheets in these workbooks are programmed in native Excel, without macros or named ranges, which means that they can be easily and safely copied into other Excel workbooks while retaining their full functionality. [Click here for details](#).

[Frontline Systems'](#) Analytic Solver, RASON, and Solver SDK. Analytic Solver V2021.5 brings metalog distributions to the fore, with a powerful new facility to automatically fit user data to the full range of possible (bounded and unbounded, multi-term) metalog distributions and to compare metalog distributions with classical distributions based on user-selected goodness of fit criteria. This release also includes a [metalog\\_panel](#) to aid choosing among metalogs with different numbers of terms.

Lumina Decision Systems' Analytica. [Lumina Decision Systems](#), which provides software to help bring clarity to difficult decisions, includes the complete metalog system ([as originally published](#)) in the recent release of its flagship product [Analytica 5.0](#). For documentation of Analytica's metalog implementation, [click here](#).

[BayesFusion's](#) GeNIe, which offers modeling software for graphical models, such as Bayesian networks, influence diagrams, dynamic Bayesian networks, unrestricted hybrid (discrete and continuous) Bayesian networks, includes the complete Metalog system (as originally published) in the recent release of its main software product GeNIe 4.0. [Click here](#) for documentation of GeNIe and its metalog implementation.

R. A CRAN approved [rmetalog\\_package is available](#). For questions and feedback, [contact Isaac Faber](#).

Python. Two Python packages are available. [Pymetalog](#) closely mirrors the R package. [Metalogistic](#) is coded so as to take full advantage of the SciPy platform of operations on continuous probability distributions.

SIPmath Modeler Tools. SIPMath™ Modeler Tools is a simulation environment in Excel provided by [Probability Management](#) as a free Excel add-in. The current version of the Tools offers two implementations. One is the SPT-metalog distributions (e.g. 3-term metalogs parameterized by 10/50/90 quantiles). Here is a [tutorial](#). For the general metalogs (including more terms), we offer a separate [tutorial](#).

SmartOrg's Portfolio Navigator. [SmartOrg](#), which provides a web-based software environment to enable better decision conversations in new product development and portfolio management, includes metalogs in their new release of [Portfolio Navigator 7](#) to aid probability distribution visualization and communication.

Web browser. [MakeDistribution.com](#) allows easy experimentation with fitting metalogs and other distributions to data.

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# YouTube videos explain the demo RAW 2022 demo workbooks.

YouTube Search

The Metalog Distributions  
32 subscribers

<https://www.youtube.com/channel/UCyHZ5neKhV1mSsedzDBoqyA>

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## Getting Started with the Free Metalog Excel Workbooks ▶ PLAY ALL

You can quickly find a metalog to fit your data—typically within 30 seconds or less – with the free metalog workbooks (<http://www.metalogdistributions.com/software/excelworkbooks.html>). For...



### Metalogs Workbook

The Metalog Distributions  
29 views • 1 month ago

### ELD Single Data Set Workbook

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### ELD Multi Data Set Workbook

The Metalog Distributions  
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# The Metalog Distributions: Future of Risk Management Quantification

## *RAW 2022 Agenda*

~ minutes

1. Introduction: Why metalog distributions for risk management 10

2. Practical Uses 35

- Representing empirical data
- Representing simulated data
- Representing expert-elicited data
- Combining the opinion of multiple experts
- Bayesian updating with new data

3. Using metalog distributions in your models 15

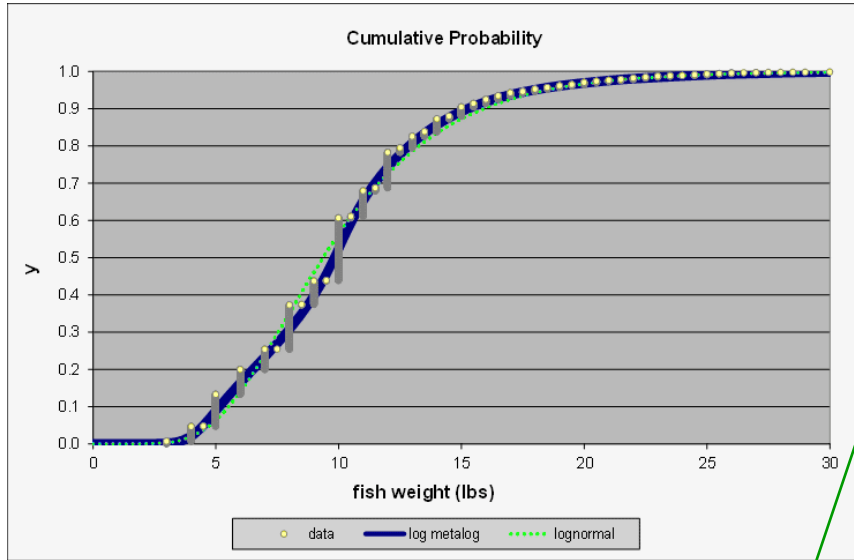
- Bringing metalogs into your modelling environment
- Simulating with metalogs
- Retrieving the metalog equations

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60



# Empirical data: fish biology

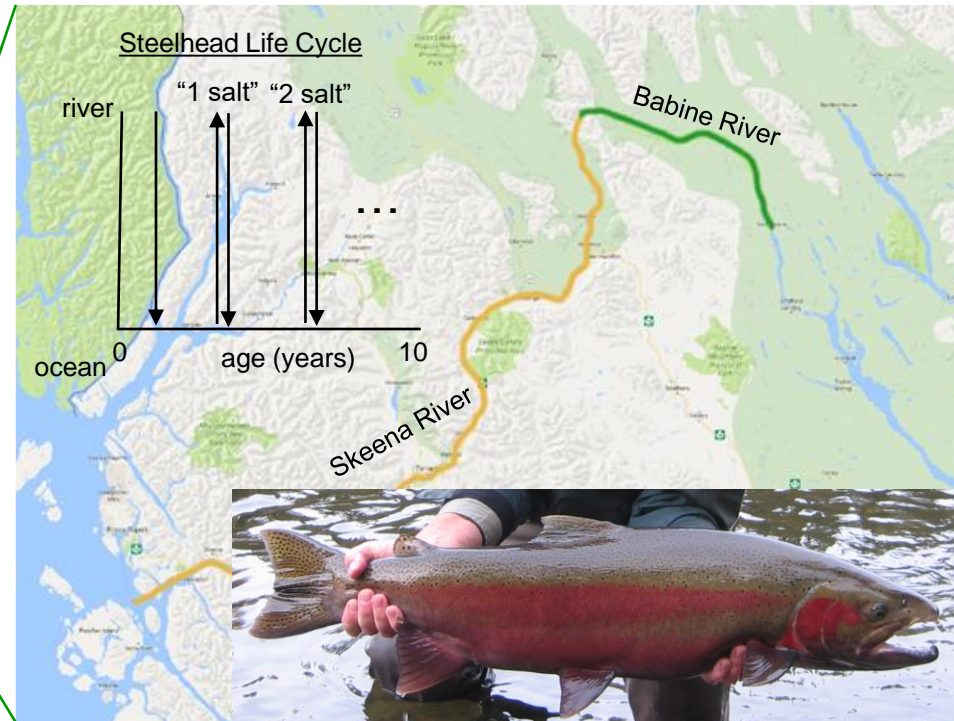


## Steelhead Trout Weight (lbs)

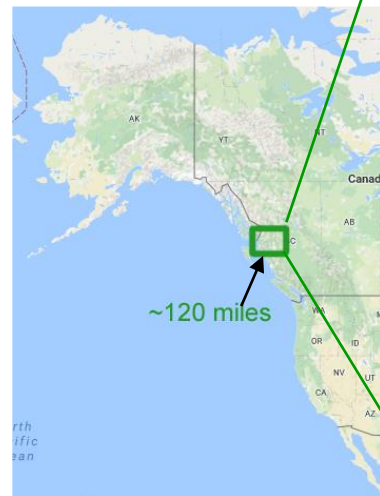


3,474 catch-and-release fish records 2010-2014. Babine River, British Columbia.

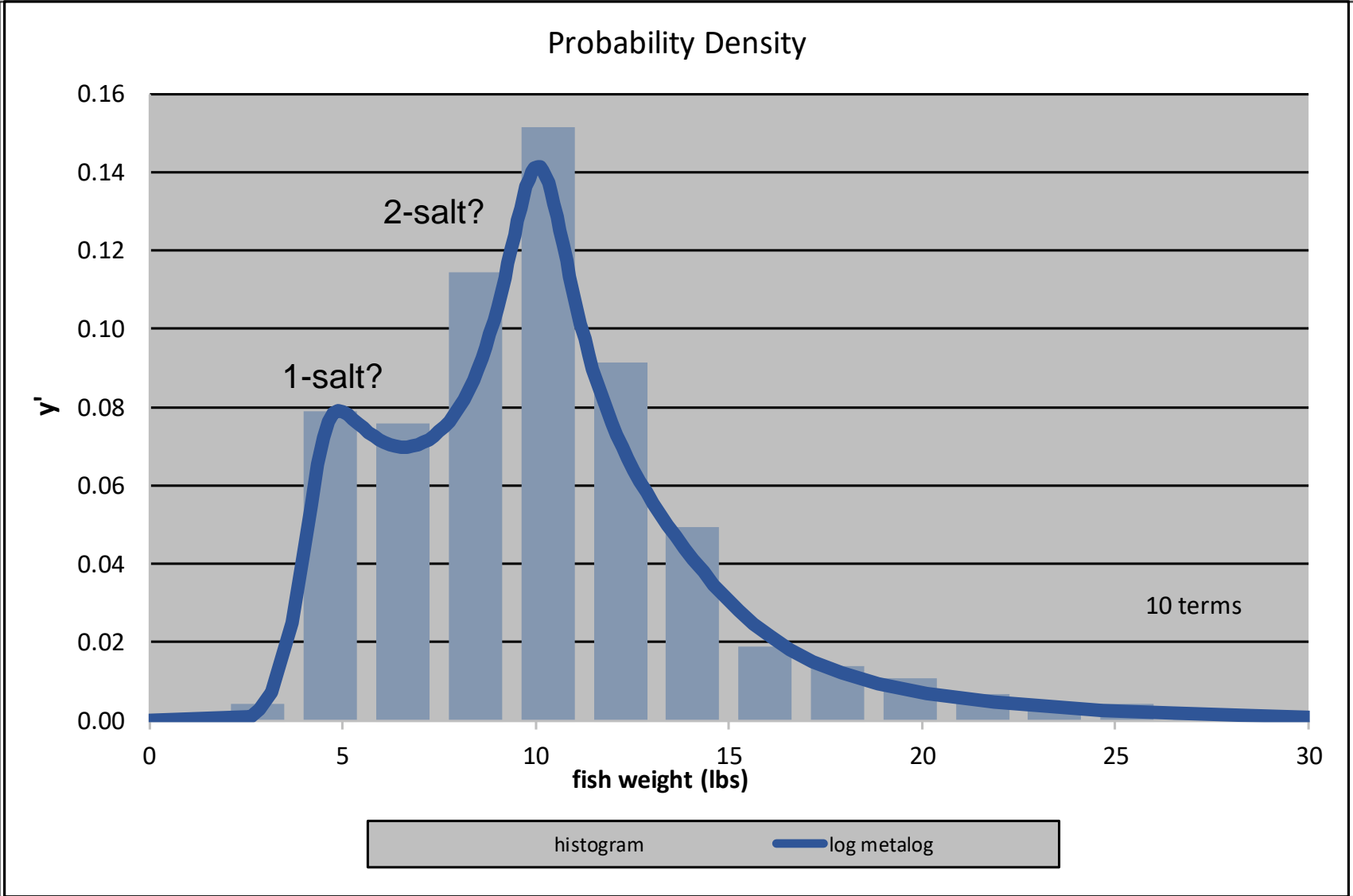
“1 salt” vs. “2 salt” fish-biology research questions:  
fish weights (relative and absolute)?  
relative population sizes?



Steelhead Lodge



# Metalogs take on the shape of the data.



AutoSave  Off ELDMetalogs\_v11\_multi\_RAW2022 • Saved Search (Alt+Q)

File Home Insert Page Layout Formulas Data Review View Developer Add-ins Help Analytic Solver Data Mining

D8

**2** Metalog distributions. Prepared by Thomas Keelin, October, 2021.

**3** This is a multi-data set ELD metalogs workbook. It's functionality is the same as the ELD metalogs workbook, except that up to ten data sets may be entered in columns C-L starting in row 51. In C10, specify the data set

**4** boundedness and lower and upper bounds, you may optionally use overrides to your base-case entries for these items in C59:L61. This workbook is licensed under the Creative Commons Attribution 4.0. International L

**5** and www.metalogs.org, you may copy it, make derivative works, and use it freely, including for commercial purposes. This workbook contains no macros or named ranges. This worksheet may be safely copied into other

**6** clicking "ELD metalog" sheet name at bottom. While we believe all calculations are correct, we assume no liability for errors.

**7** light yellow: required inputs light blue: optional inputs

<b>10</b>	data set	1
<b>11</b>	source	Tom Keelin
<b>12</b>	field	probability
<b>13</b>	short name	normal samples
<b>14</b>	description	100 samples from standard normal distribution
<b>15</b>		
<b>21</b>	metalog specifications	
<b>22</b>	m	n terms
<b>23</b>	data	for display
<b>24</b>	(2-10000)	(2-16)
<b>25</b>	100	4

**normal samples cumulative**

**messages**

(none)

**metalog panel**

<b>n=2</b>	<b>n=3</b>
<b>n=5</b>	<b>n=6</b>



# The Metalog Distributions: Future of Risk Management Quantification

## *RAW 2022 Agenda*

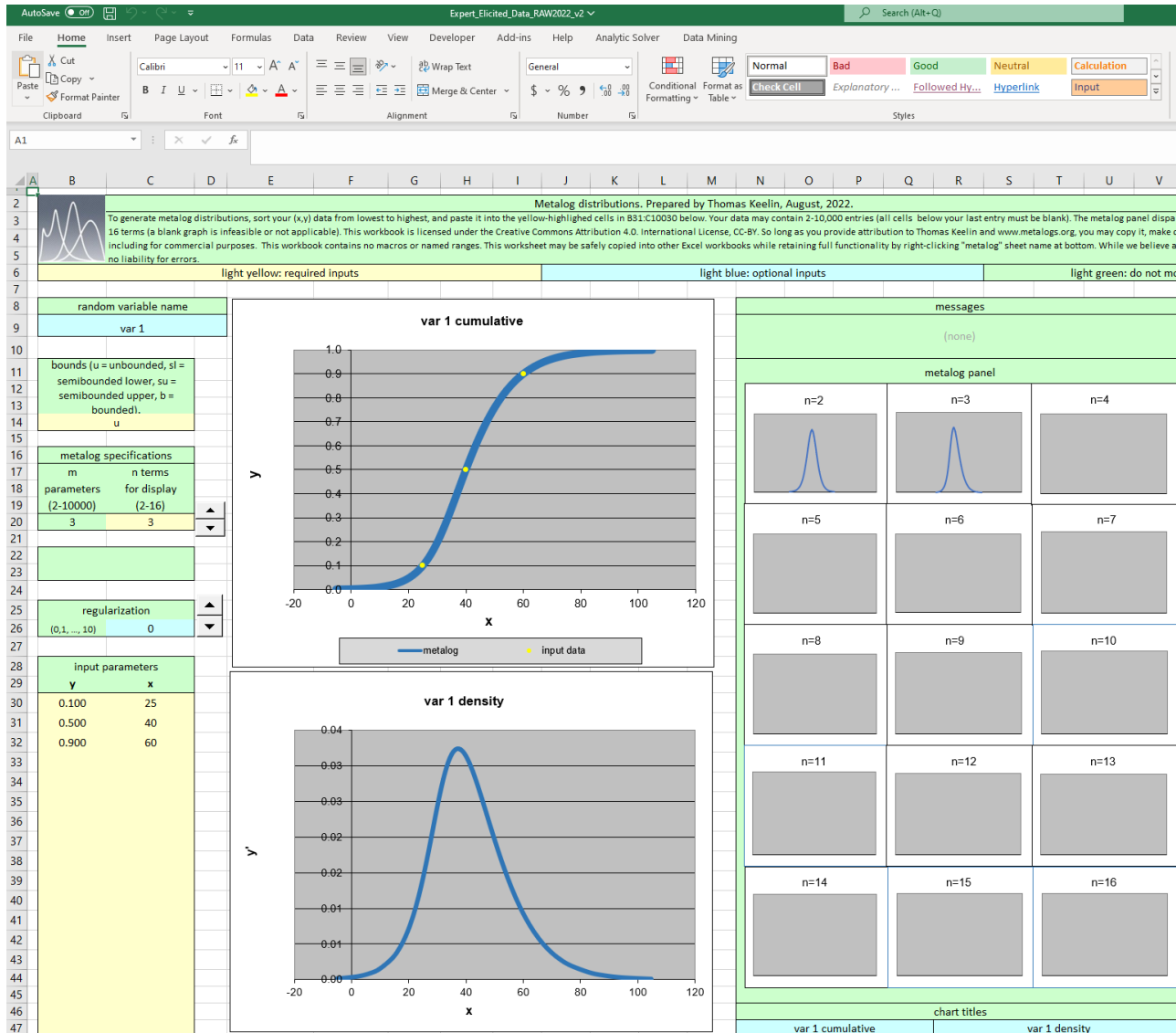
~ minutes

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---

60

# [metalogdistributions.com/demos.html](http://metalogdistributions.com/demos.html)



- “Expert-Elicited Data”
- “SPT (3 term) Metalogs”

# The Metalog Distributions: Future of Risk Management Quantification

## *RAW 2022 Agenda*

~ minutes

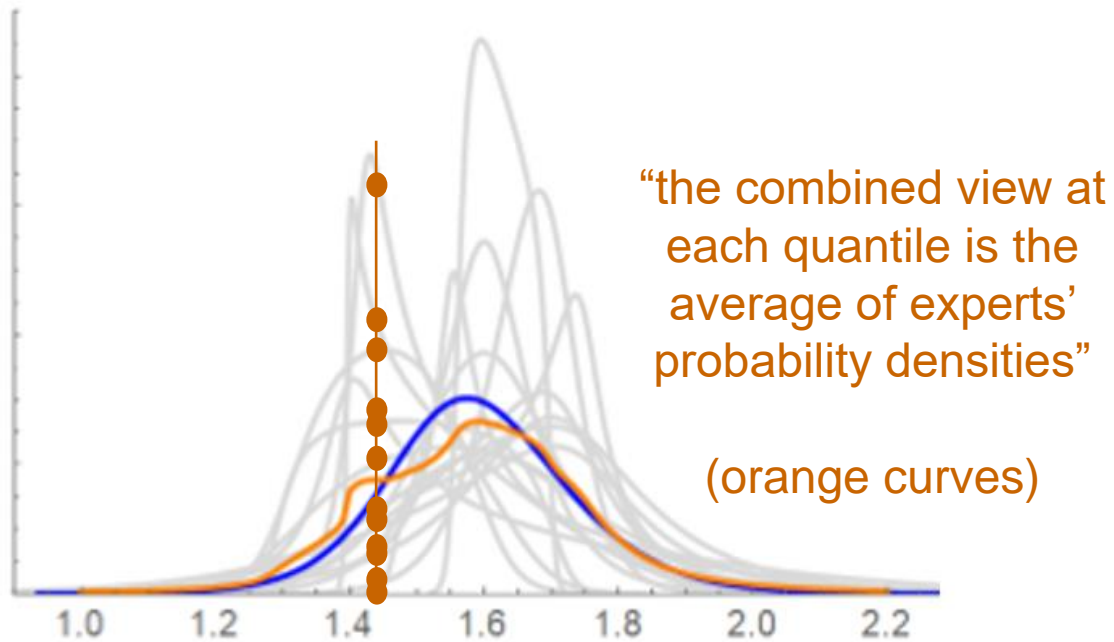
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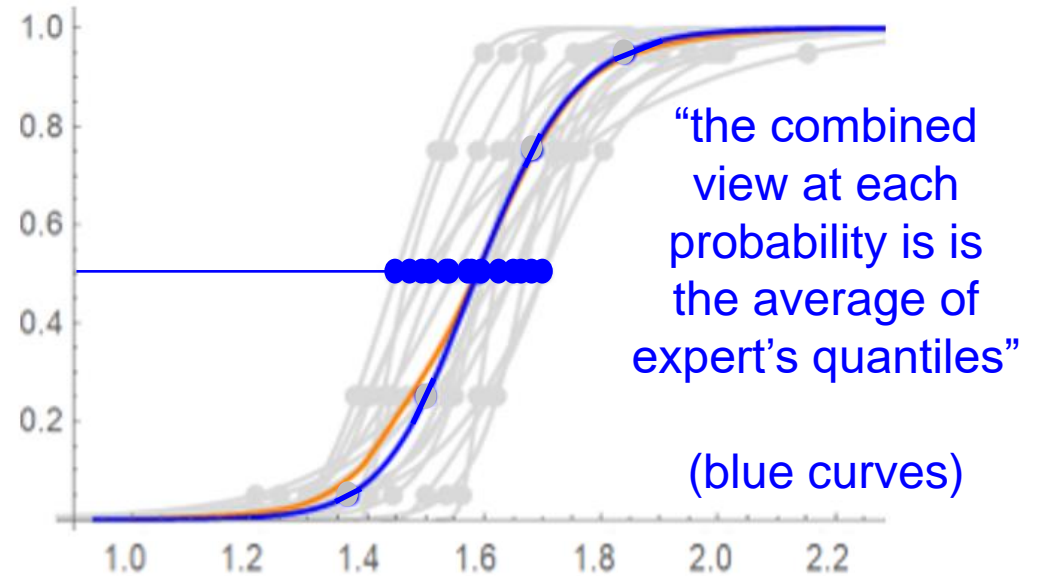
# One can combine expert opinion with metalogs by averaging quantile parameters.

averaging probabilities



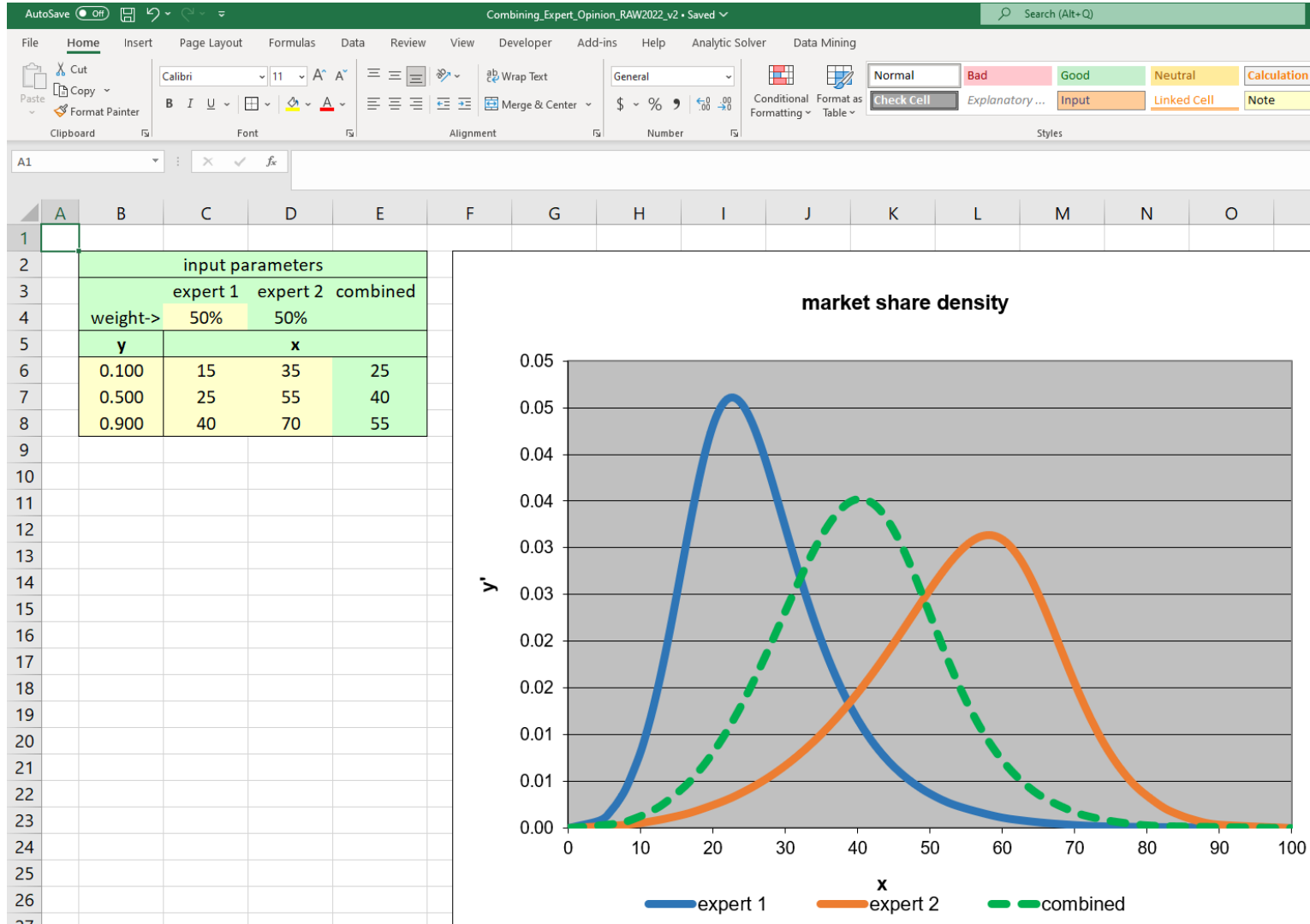
“linear opinion pooling”

averaging quantiles



“Vincenzation”

“Is It Better to Average Probabilities or Quantiles?” *Lichtendahl, Grushka-Cockayne, and Winkler. Management Science, July, 2013. Overall conclusion: “Averaging quantiles for a given probability is as good or better than averaging probabilities as a method for aggregating probability forecasts.”*



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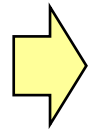
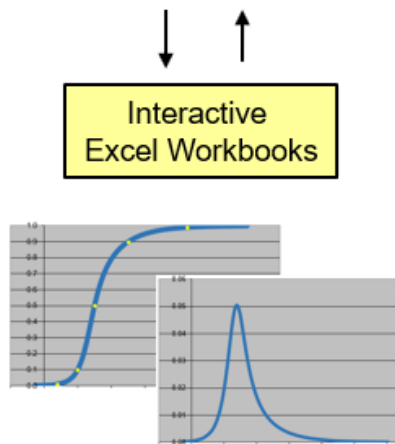
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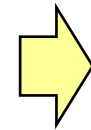
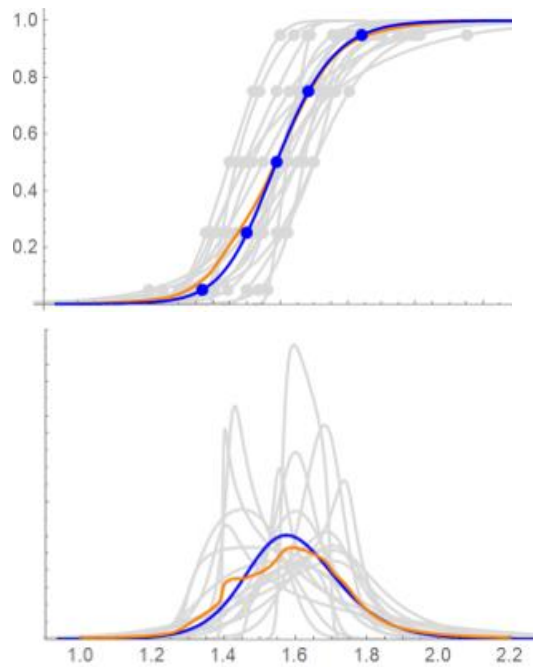
# Innovations in Eliciting, Representing, and Combining Expert Opinion and in Subsequent Bayesian Updating ...

## Eliciting and Representing Expert Opinion

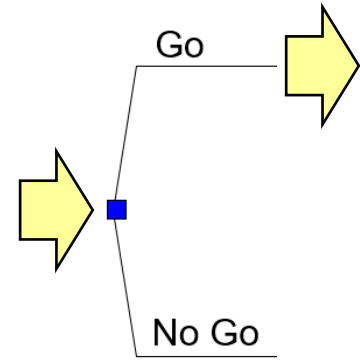
Quantile elicitation with real time feedback



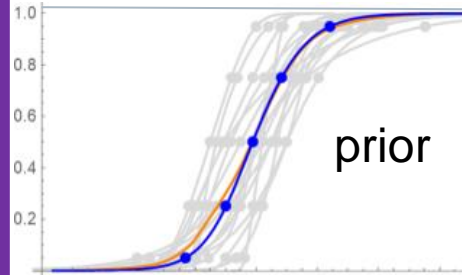
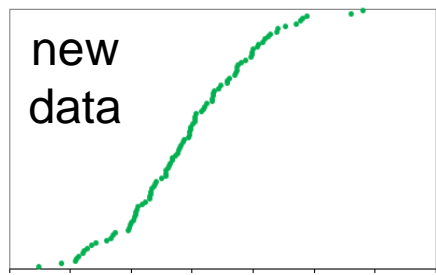
## Combining Experts' Opinion



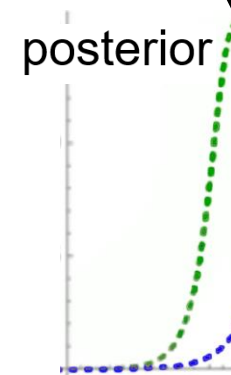
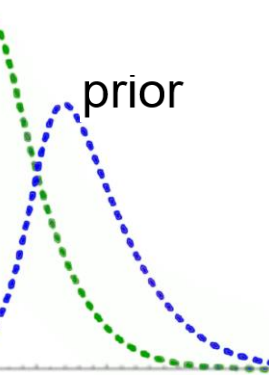
## Investment Decision Analysis



### Bayesian Updating

Bayes' rule

... *all* with Continuous Probability Distributions in Closed Form

## Montana fly fisherman meets the Williamson River, Oregon

Big Hole River, Montana



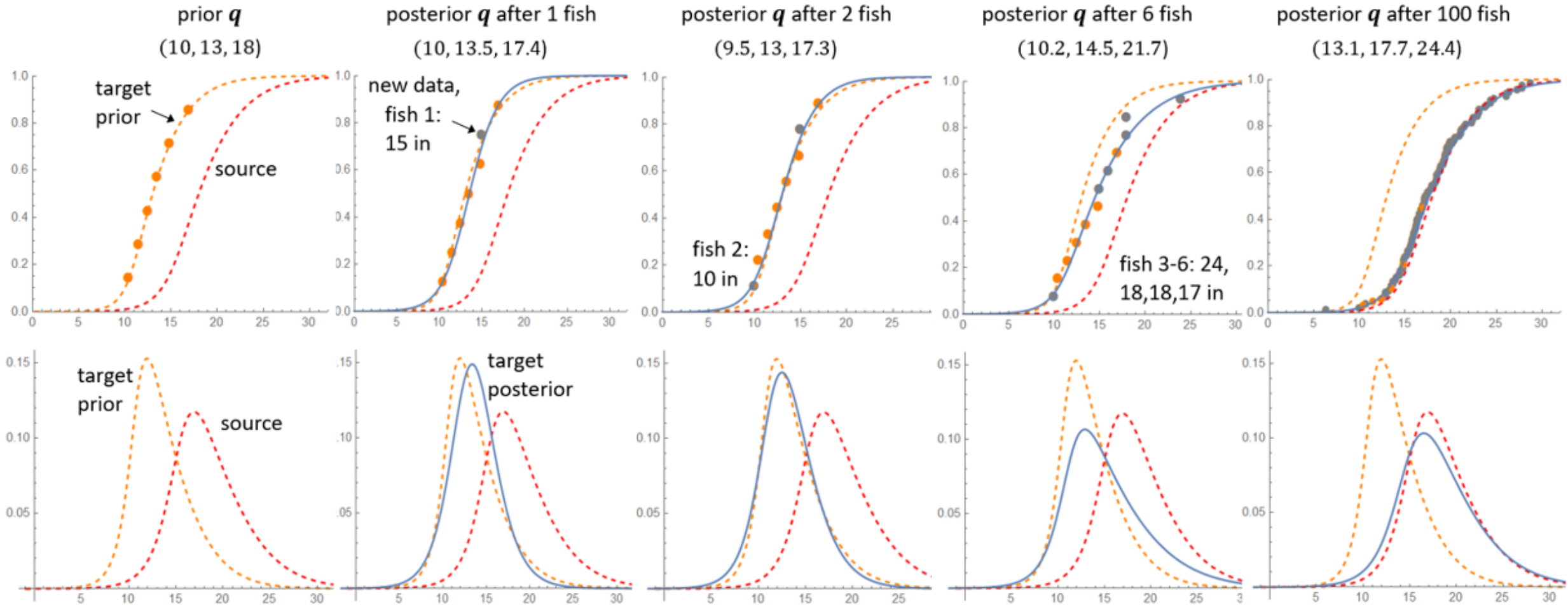
Williamson River, Oregon



- Norman is an experienced catch-and-release Montana fly fisherman
- Norman plans to try the Williamson River.
- Norman does not know that Williamson typically produces larger trout than famous Montana Rivers.
- How should he update his prior probability distribution over the size of his next trout on the Williamson based on the size of each trout he catches?



**With each Williamson trout he catches, Norman can update his distribution over the length of his next trout in closed form according to Bayes theorem.**



# The updating methodology is closed-form and published in preprint.

Full preprint explanation here: <https://osf.io/xdg5e/>

Updating methodology can be as simple as

- Combining prior and new data
- Least squares fitting of a metalog to the combined data



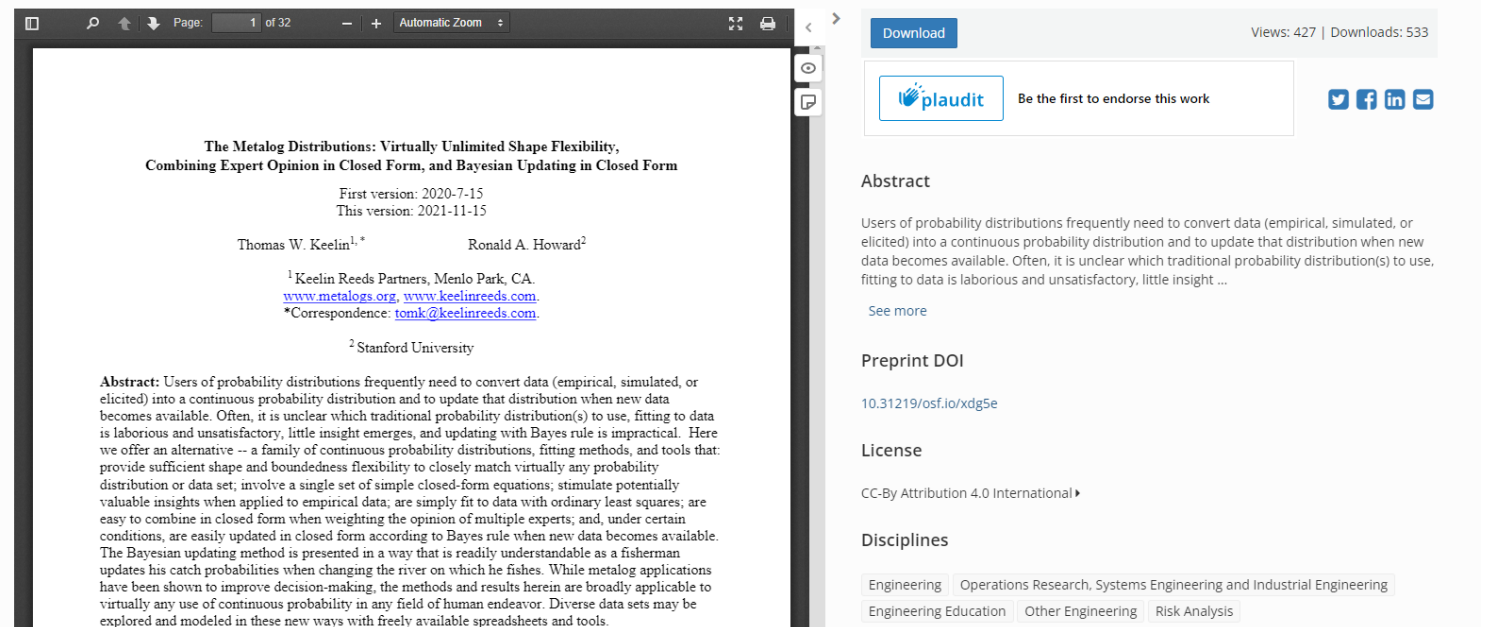
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## The Metalog Distributions: Virtually Unlimited Shape Flexibility, Combining Expert Opinion in Closed Form, and Bayesian Updating in Closed Form

AUTHORS  
Thomas W. Keelin, Ronald A. Howard

AUTHOR ASSERTIONS  
Conflict of Interest: No Public Data: Not applicable Preregistration: Not applicable



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### The Metalog Distributions: Virtually Unlimited Shape Flexibility, Combining Expert Opinion in Closed Form, and Bayesian Updating in Closed Form

First version: 2020-7-15  
This version: 2021-11-15

Thomas W. Keelin<sup>1,\*</sup> Ronald A. Howard<sup>2</sup>

<sup>1</sup>Keelin Reeds Partners, Menlo Park, CA.  
[www.metalogs.org](http://www.metalogs.org), [www.keelinreeds.com](http://www.keelinreeds.com).  
\*Correspondence: [tomk@keelinreeds.com](mailto:tomk@keelinreeds.com).

<sup>2</sup>Stanford University

**Abstract:** Users of probability distributions frequently need to convert data (empirical, simulated, or elicited) into a continuous probability distribution and to update that distribution when new data becomes available. Often, it is unclear which traditional probability distribution(s) to use, fitting to data is laborious and unsatisfactory, little insight emerges, and updating with Bayes rule is impractical. Here we offer an alternative -- a family of continuous probability distributions, fitting methods, and tools that provide sufficient shape and boundedness flexibility to closely match virtually any probability distribution or data set, involve a single set of simple closed-form equations, stimulate potentially valuable insights when applied to empirical data, are simply fit to data with ordinary least squares, are easy to combine in closed form when weighting the opinion of multiple experts; and, under certain conditions, are easily updated in closed form according to Bayes rule when new data becomes available. The Bayesian updating method is presented in a way that is readily understandable as a fisherman updates his catch probabilities when changing the river on which he fishes. While metalog applications have been shown to improve decision-making, the methods and results herein are broadly applicable to virtually any use of continuous probability in any field of human endeavor. Diverse data sets may be explored and modeled in these new ways with freely available spreadsheets and tools.

**Abstract**  
Users of probability distributions frequently need to convert data (empirical, simulated, or elicited) into a continuous probability distribution and to update that distribution when new data becomes available. Often, it is unclear which traditional probability distribution(s) to use, fitting to data is laborious and unsatisfactory, little insight ...  
See more

**Preprint DOI**  
[10.31219/osf.io/xdg5e](https://doi.org/10.31219/osf.io/xdg5e)

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**Disciplines**  
Engineering Operations Research, Systems Engineering and Industrial Engineering  
Engineering Education Other Engineering Risk Analysis

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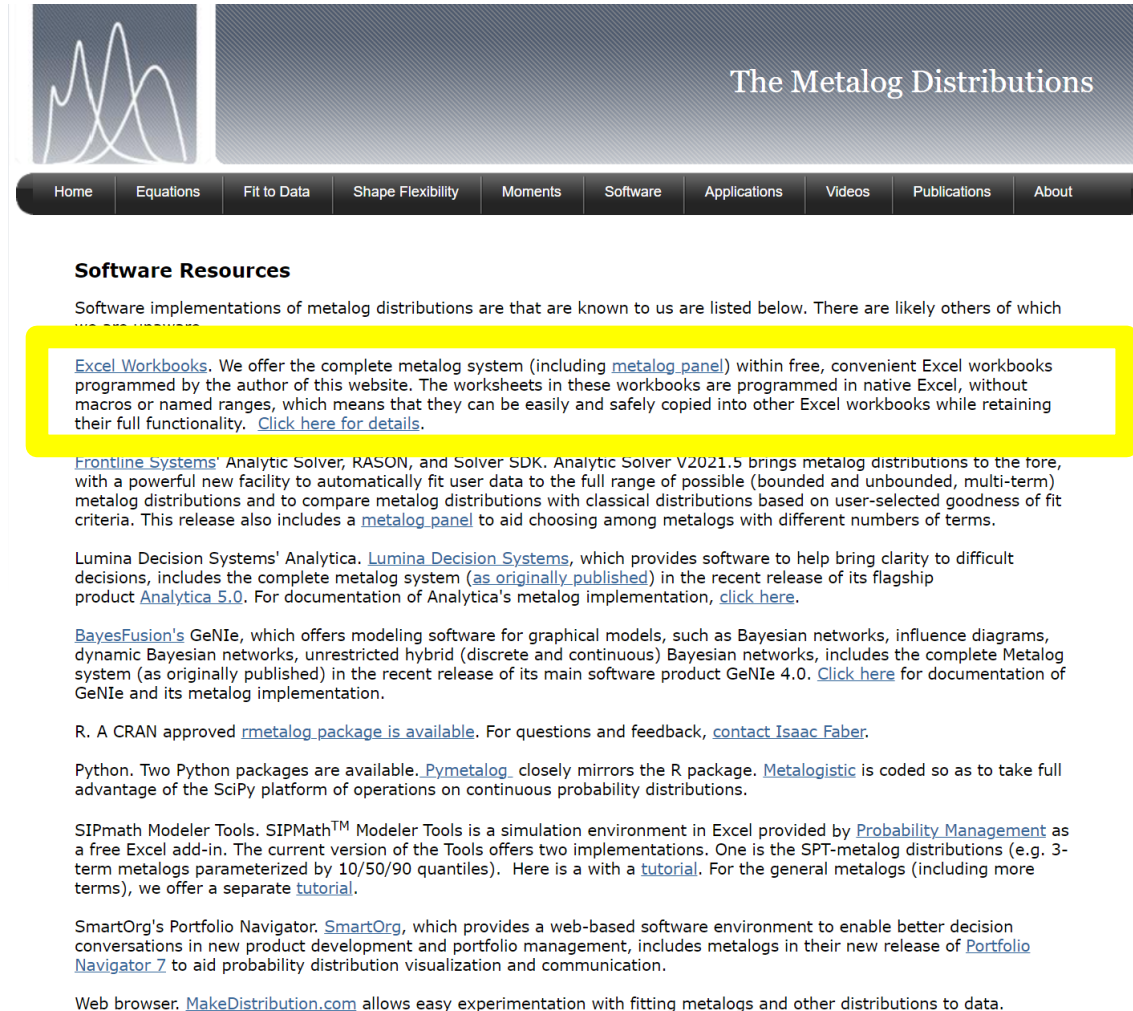
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# Using metalog distributions in your models: demo workbooks.

<http://metalogdistributions.com/software.html>

[metalogdistributions.com/demos.html](http://metalogdistributions.com/demos.html)



**The Metalog Distributions**

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### Software Resources

Software implementations of metalog distributions that are known to us are listed below. There are likely others of which we are unaware.

**Excel Workbooks.** We offer the complete metalog system (including [metalog panel](#)) within free, convenient Excel workbooks programmed by the author of this website. The worksheets in these workbooks are programmed in native Excel, without macros or named ranges, which means that they can be easily and safely copied into other Excel workbooks while retaining their full functionality. [Click here for details.](#)

**Frontline Systems' Analytic Solver, RASON, and Solver SDK.** Analytic Solver V2021.5 brings metalog distributions to the fore, with a powerful new facility to automatically fit user data to the full range of possible (bounded and unbounded, multi-term) metalog distributions and to compare metalog distributions with classical distributions based on user-selected goodness of fit criteria. This release also includes a [metalog panel](#) to aid choosing among metalogs with different numbers of terms.

**Lumina Decision Systems' Analytica.** [Lumina Decision Systems](#), which provides software to help bring clarity to difficult decisions, includes the complete metalog system ([as originally published](#)) in the recent release of its flagship product [Analytica 5.0](#). For documentation of Analytica's metalog implementation, [click here](#).

**BayesFusion's GeNIe,** which offers modeling software for graphical models, such as Bayesian networks, influence diagrams, dynamic Bayesian networks, unrestricted hybrid (discrete and continuous) Bayesian networks, includes the complete Metalog system (as originally published) in the recent release of its main software product GeNIe 4.0. [Click here](#) for documentation of GeNIe and its metalog implementation.

**R.** A CRAN approved [metalog package is available](#). For questions and feedback, [contact Isaac Faber](#).

**Python.** Two Python packages are available. [Pymetalog](#) closely mirrors the R package. [Metalogistic](#) is coded so as to take full advantage of the SciPy platform of operations on continuous probability distributions.

**SIPmath Modeler Tools.** SIPMath™ Modeler Tools is a simulation environment in Excel provided by [Probability Management](#) as a free Excel add-in. The current version of the Tools offers two implementations. One is the SPT-metalog distributions (e.g. 3-term metalogs parameterized by 10/50/90 quantiles). Here is a with a [tutorial](#). For the general metalogs (including more terms), we offer a separate [tutorial](#).

**SmartOrg's Portfolio Navigator.** [SmartOrg](#), which provides a web-based software environment to enable better decision conversations in new product development and portfolio management, includes metalogs in their new release of [Portfolio Navigator 7](#) to aid probability distribution visualization and communication.

**Web browser.** [MakeDistribution.com](#) allows easy experimentation with fitting metalogs and other distributions to data.

“Expert-Elicited Data”

“Combining Expert Opinion”

“Equation-Maker”

# The Metalog Distributions: Future of Risk Management Quantification

## *RAW 2022 Agenda*

~ minutes

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  - Representing simulated data
  - Representing expert-elicited data
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3. Using metalog distributions in your models 15
  - Bringing metalogs into your modelling environment
  - Simulating with metalogs
  - Retrieving the metalog equations

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# **The Metalog Distributions: Future of Risk Management Quantification**

# **Thank you!**

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